

AI & Teaching Workshop

HMC Math Department Retreat

2024-08-21

About Me

- Originally from Connecticut
- Undergrad at LeTourneau Univ. in Longview, TX Engineering with an Electrical Concentration (2010-2014)
- Masters and PhD in EE at Caltech (2014-2019)
- Harvey Mudd (2019-Present)



Workshop Resources

URL: <https://tinyurl.com/muddmathai>



QR Code

What tools are part of your classroom community?

- a. Chalkboard
- b. Whiteboard
- c. Slides
- d. Tablet
- e. Think-pair-share

What are the values embedded in these tools?

- a. Chalkboard: historical context, culture, limit speed
- b. Whiteboard: cleanliness, limit speed
- c. Slides: accessibility, efficiency
- d. Tablet: accessibility, limit speed
- e. Think-pair-share: community, conversation

What are the values embedded in Generative Artificial Intelligence?

What is artificial intelligence?

Popular genAI Tools



ChatGPT (OpenAI)



Midjourney



Claude (Anthropic)



Gemini (Google)



Perplexity



Copilot (Microsoft)

Figure 1: Popular generative AI tools.

Key Terms and Concepts

Generative AI is a program capable of generating content like text, images, video, and computer code. Most genAI systems respond to a prompt entered in natural language by a user.

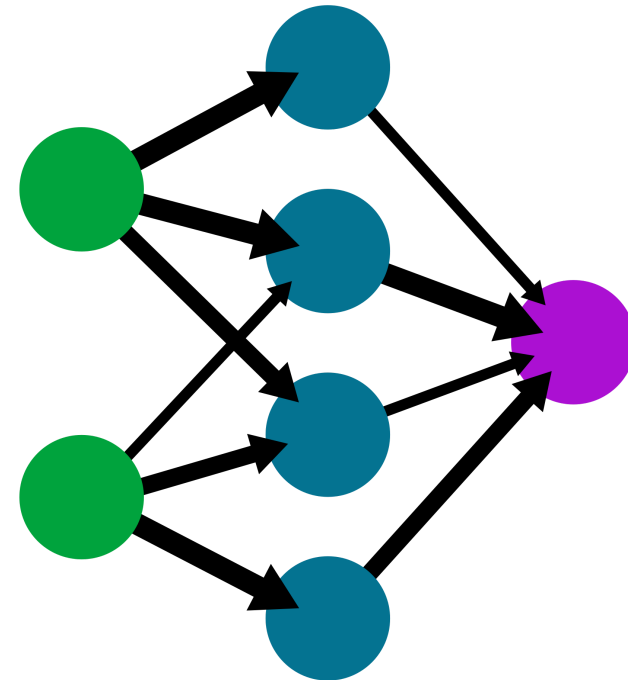
- **Large language model (LLM):** Program that behaves like very sophisticated autocomplete. Examples: [GPT-4](#), [Gemini](#).
- **Generative Pre-trained Transformer ([GPT](#)):** The specific architecture (also known as foundation model) of LLM first developed by OpenAI.
- **Multimodal:** The ability to process and generate different types of media. For example, input text and output an image or video. Examples: [Midjourney](#), [DALLE-3](#), [Sora](#).
- **Supervised machine learning:** An machine learning algorithm trained using **labeled** data.
- **Unsupervised machine learning:** A machine learning algorithm trained on **unlabeled** data.

An Abbreviated History of genAI

A rough timeline:

- 1956: Field of AI research begins at workshop at Dartmouth College.
- 1960s-1980s: Machine learning using neural networks is explored.
- Late 2000s: Development of deep learning. Deep refers to multiple hidden layers in the network.
- 2014: Generative Adversarial Networks (GANs) are developed.
- 2018: OpenAI invents the Generative Pre-trained Transformer (GPT) architecture.
- 2022: ChatGPT released to the public in November based on GPT-3.5.

A simple neural network
input layer hidden layer output layer



Discriminative AI vs. Generative AI

Discriminative models focus on classification and prediction based on given data.

- **Facial recognition:** Identify persons in photo library.
- **Speech recognition:** Siri or Alexa.
- **Medical diagnosis:** Classifying your MRI or X-ray images.
- **Autonomous navigation:** Tesla full self driving.
- **Recommendation systems:** Amazon predicting what you might want to buy.

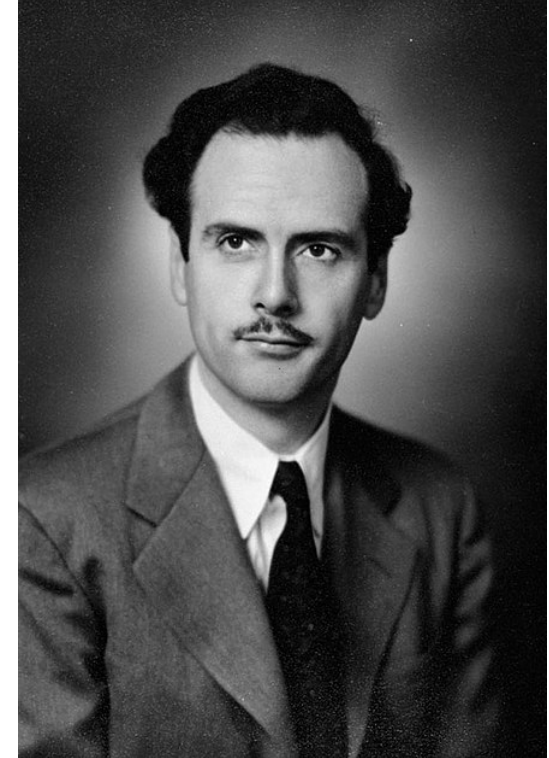
Generative models are capable of creating outputs drawn from the distribution of input data they were trained on.

- **Text generation:** LLMs which generate text from a prompt.
- **Image generation:** Midjourney or DALL-E to generate new images.
- **Drug discovery:** Alpha Fold for predicting protein structure.

The medium is the message

A technology becomes a medium as it employs a particular symbolic code, as it finds its place in a particular social setting, as it insinuates itself into economic and political contexts. A technology, in other words, is merely a machine. A medium is the social and intellectual environment a machine creates.

Understanding Media (1994)



Marshall McLuhan

Neil Postman on the message in the medium

A news show, to put it plainly, is a format for entertainment, not for education, reflection or catharsis. And we must not judge too harshly those who have framed it in this way. They are not assembling the news to be read, or broadcasting it to be heard. They are televising the news to be seen. They must follow where their medium leads. There is no conspiracy here, no lack of intelligence, only a straightforward recognition that “good television” has little to do with what is “good” about exposition or other forms of verbal communication but everything to do with what the pictorial images look like.

Amusing Ourselves to Death (2005)



Neil Postman

Whose interest?

[Y]ears from now..it will be noticed that the massive collection and speed-of-light retrieval of data have been of great value to large-scale organizations but have solved very little of importance to most people and have created at least as many problems for them as they may have solved.

Amusing Ourselves to Death (2005)

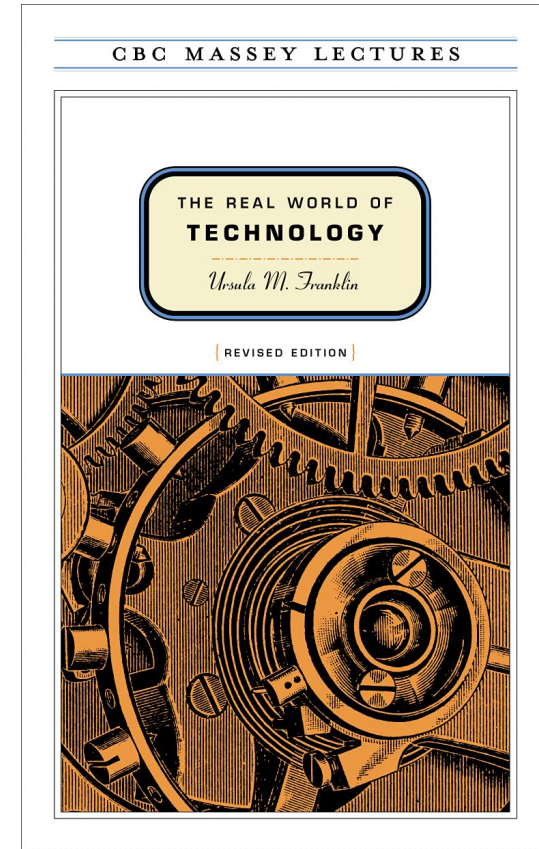


Neil Postman

What is technology?

Recently, people have become particularly interested in technology's social impact. I myself am overawed by the way in which **technology has acted to reorder and restructure social relations**, not only affecting the relations between social groups, but also the relations between nations and individuals, and between all of us and our environment. **To a new generation, many of these changed relationships appear so normal, so inevitable, that they are taken as given and are not questioned.**

The Real World of Technology (1989)



Ursula Franklin's 1989 CBC Lectures

Workshop Goals

Goals for today

By the conclusion of this workshop you will have...

- **Reflected on your values** as an educator and how they are expressed in your classes.
- **Developed a curiosity to experiment** with generative artificial intelligence.
- **Outlined text for your syllabus** to articulate your pedagogical values and examples of how those values work themselves out in your courses.
- **Developed an AI policy** for your courses.

HMC Math Dept. Goals

Practice

1. Affection
2. Application
3. Inquiry
4. Communication
5. Technology
6. Society
7. Teamwork
8. Diversity

Content

1. Breadth
2. Perspective
3. Duality
4. Proofs
5. Tools
6. Depth
7. Projects
8. Careers

Agenda for Today

Activities

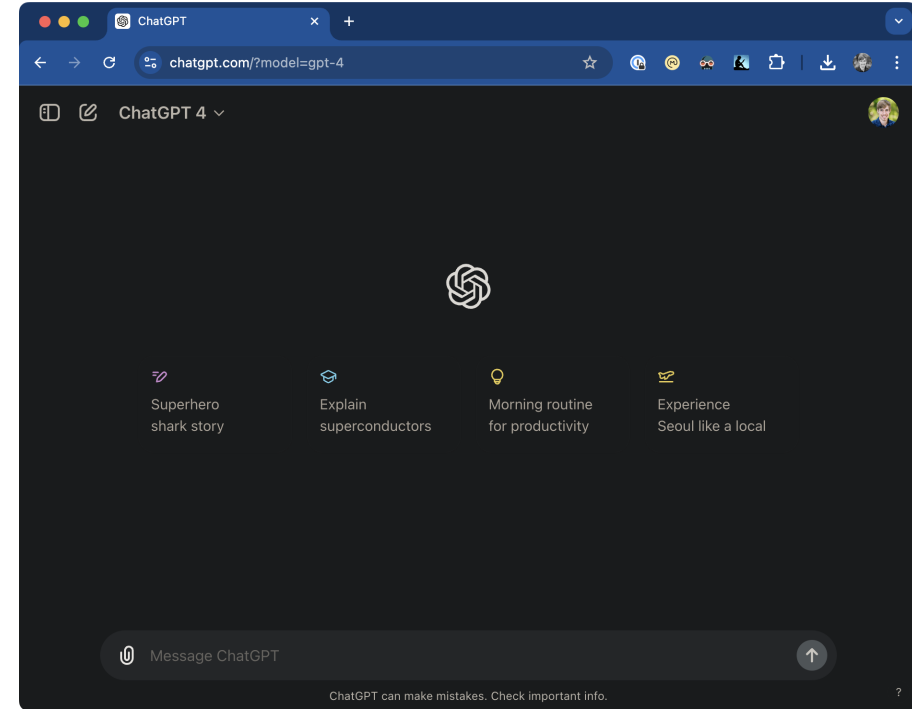
1. [GenAI Demo](#)
2. [GenAI Value Mapping](#)
3. [Syllabus Revision](#)

GenAI Demo

Lookee what he can do!



Yukon Cornelius and the Abominable Snow Monster



ChatGPT

ChatGPT the Homework Helper

HW #1 – Due Jan 30

- **Problem B.** Recall that in class, we defined a rational number $\frac{m}{n}$ to be an equivalence class of pairs (m, n) under an equivalence relation. Check that this equivalence relation is transitive: if $(p, q) \equiv (m, n)$ and $(m, n) \equiv (a, b)$, then $(p, q) \equiv (a, b)$.
- **Problem C.** We defined addition of rational numbers in terms of representatives: $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$. Show that the addition of rational numbers is well-defined.
- **Problem D.** Define a multiplication of rational numbers (corresponding to the one you are used to), and show this multiplication is well-defined. For problems B, C, D, you may assume you know properties of integers, including the cancellation law of $\mathbb{Z}$. You cannot assume that you know how to divide integers (because that is really multiplication by a rational), nor how to add or multiply rationals (because that is what you are trying to show)!
- Do also Chapter 1 (1, 2, 3a). For these problems in Rudin, you may assume that you know familiar properties of rational numbers that we were proving in Lecture 2.

Figure 2: Math 131 Homework

ChatGPT Takes Math 131

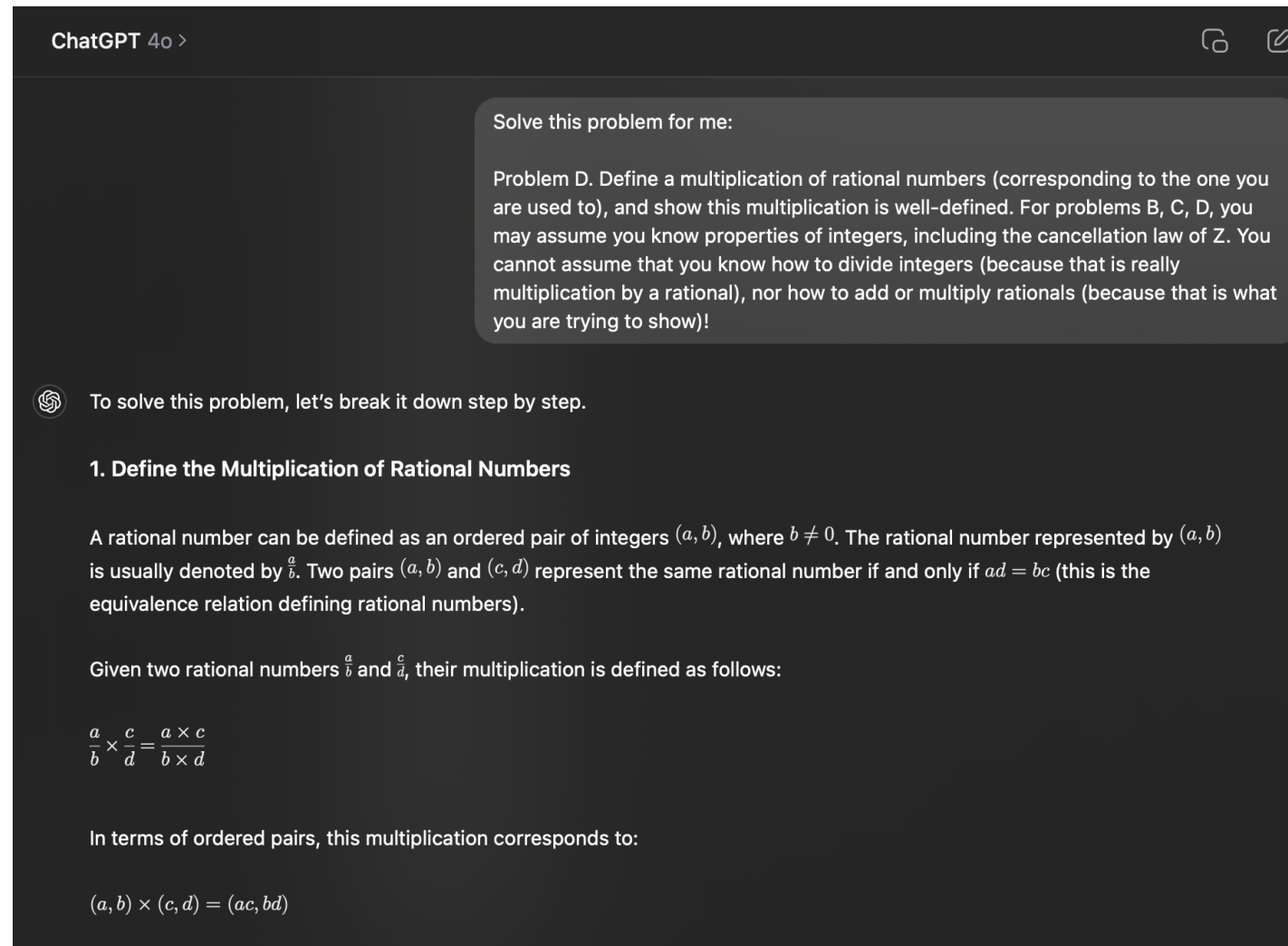


Figure 3: ChatGPT attempts Math 131

ChatGPT Takes Math 131

2. Show that Multiplication is Well-Defined

To show that this multiplication is well-defined, we need to prove that if two rational numbers are equivalent, their product with another rational number yields equivalent results.

Suppose (a_1, b_1) and (a_2, b_2) represent the same rational number, i.e., $a_1 b_2 = a_2 b_1$. Similarly, suppose (c_1, d_1) and (c_2, d_2) represent the same rational number, i.e., $c_1 d_2 = c_2 d_1$.

We need to show that:

$$(a_1 c_1, b_1 d_1) \sim (a_2 c_2, b_2 d_2)$$

This equivalence relation means we need to show:

$$(a_1 c_1)(b_2 d_2) = (a_2 c_2)(b_1 d_1)$$

Expanding both sides:

- Left-hand side: $a_1 c_1 b_2 d_2$
- Right-hand side: $a_2 c_2 b_1 d_1$

We can now use the assumptions that $a_1 b_2 = a_2 b_1$ and $c_1 d_2 = c_2 d_1$ to substitute and rewrite:

$$a_1 b_2 c_1 d_2 = a_2 b_1 c_2 d_1$$

This shows that the product of the two equivalent pairs is also equivalent, meaning the multiplication is well-defined.

3. Conclusion

The multiplication of rational numbers, as defined by $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$, is well-defined because if you multiply two pairs of equivalent rational numbers, their products will also be equivalent. This satisfies the requirements for multiplication in the set of rational numbers.

Figure 4: ChatGPT attempts Math 131

ChatGPT the Concept Explainer

What is a Taylor series?

 A Taylor series is a mathematical concept used in calculus to represent a function as an infinite sum of terms that are derived from the function's derivatives at a single point. Essentially, it is an approximation of a function around a certain point, which can provide very accurate results depending on the number of terms used in the series.

The general form of a Taylor series for a function $f(x)$ around a point a is given by:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots$$

This can also be written in summation notation as:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$$

Where:

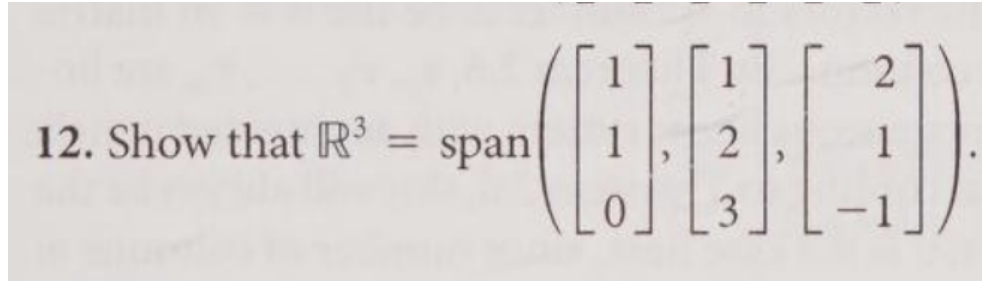
- $f^{(n)}(a)$ is the n -th derivative of the function evaluated at the point a .
- $n!$ is the factorial of n , which is the product of all positive integers up to n .

The Taylor series is particularly useful in approximating functions that may be complex to evaluate directly, especially near the point a . If the Taylor series is centered at $a = 0$, it is often called a Maclaurin series.

In practical applications, the Taylor series can be truncated to a finite number of terms to approximate the function, with the approximation becoming more accurate as more terms are included.

Figure 5: Taylor series explanation

ChatGPT the Proof Writer



12. Show that $\mathbb{R}^3 = \text{span}\left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}\right)$.

Figure 6: Math 40 proof

ChatGPT Math 40

12. Show that $\mathbb{R}^3 = \text{span}\left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}\right)$.

Prove this theorem as if you were Terence Tao

 $\mathbb{R}^3 = \text{span}\left\{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}\right\}$

To prove that $\mathbb{R}^3 = \text{span}\left\{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}\right\}$, we need to show that any arbitrary vector in $\mathbb{R}^3$ can be expressed as a linear combination of these three vectors.

Step 1: Expressing an Arbitrary Vector

Let $\mathbf{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ be any arbitrary vector in $\mathbb{R}^3$. We need to find scalars x_1 , x_2 , and x_3 such that

$$\mathbf{v} = x_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}.$$

This equation can be expanded as:

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}.$$

Thus, we obtain the following system of linear equations:

$$\begin{aligned} a &= x_1 + x_2 + 2x_3, \\ b &= x_1 + 2x_2 + x_3, \\ c &= 3x_2 - x_3. \end{aligned}$$

Figure 7: ChatGPT attempts Math 40 proof

ChatGPT Math 40

Step 2: Solve the System of Equations

We can solve this system for x_1 , x_2 , and x_3 in terms of a , b , and c .

Starting with the third equation:

$$x_3 = 3x_2 - c.$$

Substitute x_3 into the first two equations:

$$a = x_1 + x_2 + 2(3x_2 - c),$$

$$b = x_1 + 2x_2 + (3x_2 - c).$$

This simplifies to:

$$a = x_1 + 7x_2 - 2c,$$

$$b = x_1 + 5x_2 - c.$$

Now subtract the second equation from the first:

$$a - b = (x_1 + 7x_2 - 2c) - (x_1 + 5x_2 - c),$$

$$a - b = 2x_2 - c,$$

$$x_2 = \frac{a - b + c}{2}.$$

Now substitute x_2 back into the second equation for x_1 :

$$b = x_1 + 5\left(\frac{a - b + c}{2}\right) - c,$$

$$x_1 = b - \frac{5(a - b + c)}{2} + c = \frac{2b - 5a + 5b - 5c + 2c}{2},$$

$$x_1 = \frac{-5a + 7b - 3c}{2}.$$

Finally, substitute x_2 into the equation for x_3 :

$$x_3 = 3\left(\frac{a - b + c}{2}\right) - c = \frac{3a - 3b + 3c - 2c}{2} = \frac{3a - 3b + c}{2}.$$

Figure 8: ChatGPT attempts Math 40 proof

ChatGPT Math 40

Step 3: Conclusion

We've now expressed x_1 , x_2 , and x_3 in terms of a , b , and c , confirming that any vector in $\mathbb{R}^3$ can indeed be written as a linear combination of the given vectors. Hence, the set $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \right\}$ spans $\mathbb{R}^3$.

Thus, $\mathbb{R}^3 = \text{span} \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \right\}$ is proven.

Figure 9: ChatGPT attempts Math 40 proof

High-level Observations

- LLM-generated text can reproduce the type of structure and format we would like to see.
- Clean syntax is no longer a worthy proxy for clear thought.
- The types of errors in LLM-generated text are tricky to detect.
- The LLMs “solves” the blank page problem.
- Generating *a* solution is now effectively free.
- The floor for communicating mathematical thinking has been raised.

Activity #1: GenAI Value Mapping

Exploring alignment

GenAI Value Mapping Exercise

For each of the following values, rank how significantly you anticipate generative AI will impact it (the inner product, if you'll humor me). Then jot down a few notes on how you expect generative AI to have either a positive or negative impact.

Value	genAI · Value	Comments
P1: Affection	-2 -1 0 1 2	
P2: Application	-2 -1 0 1 2	
P3: Inquiry	-2 -1 0 1 2	
P4: Communication	-2 -1 0 1 2	

Course intersections

[illegible]

Experimenting with AI in your courses

Some Principles

1. Build agency
2. Encourage reflection
3. Explain what learning looks like
4. Make values and learning goals explicit
5. Find ways to show your students you trust them
6. Build socio-technical thinking

Prototypes

For Students

1. Homework Helper
2. Interactive Encyclopedia
3. Ideation Partner

For Instructors

1. Pedagogy Coach
2. Active Learning Activity Generation
3. Feedback Summarization

Activity #2: Syllabus Revision

Elements of a Syllabus Policy

1. A brief explanation of what generative AI is (here's a nice [interactive demo](#) of transformers from the Financial Times)
2. Your policy on generative AI use in the classroom
 - Avoid
 - Embrace
 - Critical Exploration
3. An explanation of the reasoning behind your policy
4. An articulation of your expectations around use if you are allowing genAI to be used (e.g., norms around disclosure, reporting requirements, etc.)

Resources

<https://bit.ly/AI-Syllabi>

 Syllabi Policies for Generative AI ☆   File Edit View Insert Format Data Tools Extensions Help						
Q Menus   100%  View only						
A1	Course					
	A	B	C	D	E	F
1	Course	Discipline	Policy in the Syllabus	Contributor	Rights for Reuse	Institution
2	ACM271 - Intro to Arts & Cultural Management	Arts	Use of generative AI in class and for assignments You are welcome to use generative AI tools (e.g. ChatGPT, Dall-e, etc.) in this class as doing so aligns with the course objectives of Creating Mission and Vision Statements, Creating a Sample Budget, and Creating Plans for your own Performing Arts Organization or Museum. You may not use generative AI for the remaining assignments. You are responsible for the information you submit.	Max Evjen Twitter: @cantus94 Bluesky: @cantus94.bsky.social		Michigan State University
3	CD115 Design Studio 1	Arts	Submit your own work. If you use a source for support, include quotes and a citation. Academic dishonesty includes taking content from an Internet search, another person/entity, or AI technology such as ChatGPT (either directly or with modification) and representing it as your answer.	Susan Leopold		Fashion Institute of Technology
4	Digital Design Studio	Arts	Use of AI such as Chat GPT: This is somewhat new territory at academic institutions and policies are being developed to address the potential negative impact of AI, which might lead to acts of plagiarism as well as diminish learning overall. In this course all your work must be your own, including the small bits of writing that are part of the course project criteria. In an ideal world, Chat GPT can be/become an excellent	Maura Jurgrau		Fashion Institute of Technology
5	BIOSC_3750 General Microbiology	Biology	Ethical Use of Generative AI in this Course Generative AI (GAI) tools, such as ChatGPT, Google Gemini, DALL-E 2, and others, are transforming various aspects of society, including academic and scientific fields. In this course, you may utilize GAI as an editor, translator, idea generator, data visualization tool, or tutor. GAI can be a powerful learning tool.	Caroline Grunenwald	Contact me for specific permissions	University of Missouri
6	Business	Business	I expect you to use AI (ChatGPT and image generation tools, at a minimum), in this class. In fact, some assignments will require it. Learning to use AI is an emerging skill, and I provide tutorials in Canvas about how to use them. I am happy to meet and help with these tools during office hours or after class. Be aware of the limits of ChatGPT.	Ethan Mollick		Wharton School University of Pennsylvania

Closing Thoughts

Some things you should discuss with students

- When to use an LLM vs. talk to your classmate or instructor
- The environmental considerations: the computational horsepower to run these things takes a lot of electricity and water.
- Data privacy concerns: The ethical issues surrounding training data
- Embedded biases

Contact Me

If you have questions or comments, please connect!



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Megan J. Robinson, [Megan J. Robinson](#)

"Awesome thoughts on teaching, learning and growth in higher education, through the lens of experimentation and prototyping."

Mary Chris Escobar, [The Healthier Hustle](#)

"Thoughtful essays on the social impacts of technology from someone who builds and teaches technology."

Rob Nelson, [AL Log](#)